RELATIONSHIP BETWEEN CYCLIC UNDRAINED
TRIAXIAL STRENGTH OF SANDY SOIL
AND GEOTECHNICAL INDEX VALUES

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Abstract

(Objectives)

The methods for finding and comparing earthquake induced shear stress and cyclic
undrained shear strength of the ground are well known. These values are used to evaluate
the liquefaction potential of the ground. Of the various methods available for determining
these values, those considered to be the most accurate are the earthquake ground response
analysis method for earthquake induced shear stress, and the cyclic undrained triaxial test,
conducted in the laboratory to determine cyclic undrained shear strength of the ground.
The objective of this paper is to present the process by which the authors have worked out an
experimental formula that is very useful in the evaluation of cyclic undrained triaxial strength
(hereafter referred to as $R_{to}$) on the basis of other geotechnical index values. The authors' new
formula is the product of data derived from numerous cyclic undrained triaxial tests.
The formula was worked out by applying the following multiple regression analysis method:

$$R_{to} = \beta_0 + \sum \beta_i X_i$$

where $\beta_0$ = intercept, $\beta_i$ = multiple regression coefficient, $X_i$ = geotechnical index values.

(Summary)

An axial loading type cyclic triaxial apparatus was used for the experiments.
Figures 1 and 2 show a schematic view of the apparatus and a detailed diagram of the cell
respectively. Undisturbed samples for use in the experiment were taken from alluvial sandy
deposits and sandy reclaimed ground layers by means of sand sampling. All samples were
frozen immediately after collection. Table 1 is the list of the whole samples used for cyclic
triaxial testing.

Figure 3 shows some typical wave form records produced in the experiments.
Basically the records taken in the experiments were of 3 types: cyclic axial loading, cyclic axial
displacement and excess pore water pressure.

Figure 4 is a graphic definition of $R_{to}$. It defines $R_{to}$ as the stress ratio (stress
ratio is the ratio of 1/2 the single amplitude of cyclic axial stress and initial effective confining
pressure) obtained when a sinusoidal axial wave is applied to the sample 20 times, making
axial double amplitude of displacement equivalent to 5% strain.

At first, the authors considered effective overburden pressure $\sigma'_v$, dry density $\gamma_d$,
relative density Dr, mean diameter $D_{50}$, 10% diameter $D_{10}$, fine content FC and N-value (which
is defined as the number of blow counts in the standard penetration test) as possible geotechnical
index values that could be used to indicate $R_{to}$. However, because of problems in
obtaining and evaluating data, it was finally decided to use only $\sigma'_v$, N-value, $D_{50}$ and FC.

Altogether, 142 samples were taken. At each given depth, 1 sample (divided into
3 or 4 specimens) was taken.
Figures 5 through 12 show the relationships between $Rl_{20}$ and various geotechnical index values. None of these graphs show good correlations. This led the authors to consider multiple regression analysis.

Figure 13 shows the correlation coefficients between the variables used in multiple regression analysis. It will be noted that a fairly close correlation exists between $D_{90}$ and $FC$, which seems to be perfectly natural. These two variables were not considered together in the multiple regression formula. Table 2, based on the above determinations, shows all possible combinations of multiple regression analysis. The multiple regression formula for $Rl_{20}$ was solved for all these combinations. For log $Rl_{20}$, the formula was solved only for three combinations that should show statistical significance. (See Fig. 14 (1)—(3) and Fig. 15)

Both $Rl_{20}$ computation results in Figure 14 and log $Rl_{20}$ computation results in Figure 15 are shown as multiple correlation coefficients and variance ratios in the two graphs.

Of these computation results, it was found that the multiple correlation coefficient whose value is highest results from combinations of log $Rl_{20}$, log $\sigma'$, $N$ and log $FC$. This is shown by the following experimental formula.

$$\log Rl_{20} = -0.785 + N \cdot \log 1.012 + 0.0868 \log FC - 0.441 \log \sigma'$$

In other words

$$Rl_{20} = 0.164 \cdot (1.012)^N \cdot (FC)^{0.0868} \cdot \sigma'^{-0.441} \ldots$$ (See formula (3) of thesis)

Figure 16 shows the relationship between the final values for $Rl_{20}$ as obtained by the formula (3) (calculated $Rl_{20}$) and those $Rl_{20}$ values actually measured (measured $Rl_{20}$). The graph also shows the relationship using existing data. The correlations can be said to be good.

Figure 17 shows the same information as Figure 16, using data obtained from the existing formula. (See formula (5) of thesis). No general pattern can be discerned in this graph. Then main difference between the formula (3) and the formula (5) lies in the nature of the dependency of both formulas on $N$-value and $\sigma'$.

Most research to date has found cyclic undrained shear strength to be greatly affected by $N$-value in clean sand. In this aspect, the formula (5) is in good agreement with the traditional view of the $N$-value and cyclic undrained shear strength relationship. However, the formula (3) can be considered superior than the formula (5) for determining cyclic undrained shear strength by sand sampling and cyclic triaxial testing. From another point of view, the formula (3) can be considered to cast doubt on the relative accuracy of the presently used systems for evaluating cyclic undrained shear strength. This area should be considered to be a subject for further study.

1 INTRODUCTION

In recent years, liquefaction of sandy ground during earthquakes has come to the fore as an important field of research. Research includes the nature of this type of ground and the design of structures to be built on it. Many methods have been proposed for predicting ground liquefaction. The methods range from quite simple ones, using only two properties such as soil grain characteristics and $N$-value and conducting computer analysis to quite complex methods. The most accurate method to date for evaluating liquefaction potential is to compares earthquake induced shear stress of the ground and cyclic undrained shear strength. The most common methods used to determine shear stress are the wave propagation theory, the lumped mass method, the finite element method, etc. These techniques are used to conduct analysis of ground response during earthquakes. For analysis of shear strength of the ground,
the most commonly used methods are cyclic triaxial testing and cyclic simple shear testing.

The object of this paper is to present the derivation of an experimental formula showing the relationship between cyclic undrained triaxial strength and four geotechnical index values, that is, N-value, mean diameter, fine content and effective overburden pressure. The actual formula was developed using multiple regression analysis. Until the present, many attempts have been made to discover the relationship between cyclic undrained triaxial strength and soil properties. All of these attempts have explored the apparent relationship between two or three variables, objectively combining these tendencies to produce a certain formula. The authors have attempted here to evaluate as objectively as possible the relationship between each variable to produce a suitable experimental formula.

## 2 TESTING APPARATUS

Figure 1 shows in outline the testing apparatus used. The specimen is placed within the cell, in which a certain confining pressure is maintained. A certain number of cyclic loads are applied in the direction of the axis, and the relationship between the number of loads applied and displacement, pore water pressure, etc., are recorded. While confining pressure is applied by means of a compressor, cyclic pressure is applied in the form of oil pressure. A semi-conductor type pressure transducer detects static pressure (confining pressure) by electric

![Diagram of Testing Apparatus](image)

Fig. 1 Schematic view of the apparatus
measuring. Cyclic load is measured with the load cell. Two LVDTs measure displacement. Pore water pressure is measured by the same type of transducer used in static pressure measurement.

Various kinds of cyclic wave forms are possible for testing, including sine waves, rectangular waves and certain random input waves.

The following information is necessary to obtain accurate experimental results:

- static pressure: $8 \text{ km/cm}^2 \text{ max}$
- cyclic loading: $\pm 250 \text{ kg max}$
- frequency: $0.1 \text{ to } 5 \text{ Hz}$

The lower right hand section of Figure 1 is a carbonic acid gas circulation system. This system is used for quickly and completely saturating sandy soil specimens.

Next, let us explain the detailed diagram of the cell in Figure 2. The cell uses 3 bellows–phragms to seal the piston section. The piston is centered by means of bearings in the central section. This type of structure permits independent control of axial and lateral pressure. Thus, subsoil stress conditions can be recreated at will. This is the outstanding feature of the cell. In other ways, it is almost the same as the ordinary triaxial cell.
3 SAMPLES USED IN TESTING

All samples used in testing consisted of sandy soil from alluvial deposits or reclaimed land areas. All undisturbed samples were obtained by twist sampler. Fine content of all samples taken amounted to as much as 60%. Table 1 summarizes the types of samples used in testing. All saturated samples taken from below ground level were frozen at the site and stored in a freezer at the laboratory. Afterwards, these samples were removed from the freezer and reduced to specimens of either 50mm diameter and 100mm length or 70mm diameter and 140mm length with the aid of a saw, file, etc. It should be noted that the diameter of the samples are predetermined by the diameter of the twist samplers, so that the main objective in cutting the specimens was to establish uniform lengths.

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4 TESTING PROCEDURE

(1) Setting of Specimen in Triaxial Cell

After trimming the specimen, diameter, length, weight, etc. are measured carefully, and the specimen is covered with a rubber membrane and set in the triaxial cell. It is important that this part of the operation be carried out quickly, before the specimen thaws.

(2) Thawing of the specimen

Next, the specimen is thawed by subjecting it to a 0.2kg/cm² vacuum for 2 hours.
(3) Saturation of the Sample

Samples in general arrive at the laboratory in a somewhat unsaturated condition. As it is important that the liquefaction test be conducted on a completely saturated specimen, this is accomplished by circulating carbonic acid gas and deaerated water in turns through the specimen, and applying an appropriate back pressure load to attain complete saturation. Since carbonic acid gas is 100 times more water-soluble than air, this process of saturation is highly efficient. Saturation is confirmed by using Skempton’s pore pressure coefficient. This coefficient is defined as follows:

$$\beta = \frac{\Delta U}{\Delta \sigma}$$  \hspace{1cm} (1)

where

$$\Delta \sigma = \text{increase of confining pressure, and}$$

$$\Delta U = \text{increase of pore water pressure in an undrain condition when confining pressure } \Delta \sigma \text{ increases. When } \beta = 1.00 \text{ the specimen is completely saturated.}$$

In the tests conducted by the authors, $\beta$ was equal to or exceeded 0.95 for all specimens.

(4) Consolidation of the Specimen

When saturation was complete, the specimens were consolidated at isotropic confining pressure that was equivalent to effective overburden pressure. Consolidation time was set at about 2 hours. Back pressure was applied during consolidation.

(5) Cyclic Undrained Shear Test

When consolidation is complete, a sine wave cyclic load is applied in the direction of the axis, with a cyclic load frequency of 0.5 Hz or 1.0 Hz. Cyclic loading, displacement and pore water pressure are automatically recorded. The test is continued until double axial strain amplitude exceeds 10%.

5 INTERPRETATION OF TEST RESULTS AND DEFINITION OF CYCLIC UNDRAINED TRIAXIAL STRENGTH

Figure 3 shows representative examples of recordings taken during testing. The purpose of the liquefaction test is to investigate the way that increases in displacement and excess pore water pressure occurs in response to cyclic pressure at a fixed amplitude. Cyclic loading, displacement and changes in pore water pressure are recorded, and cyclic undrained triaxial strength is calculated. Cyclic undrained triaxial strength tells us the relationship between stress ratio (the ratio of 1/2 the single amplitude of cyclic axial stress and initial effective confining pressure) and the number of cyclic loadings necessary for liquefaction to occur (the authors refer to this number as $N_l$). Stress ratio is easily determined on the basis of cyclic load, sectional area of the specimen and effective confining stress. There are various methods for determining $N_l$. Here, $N_l$ is defined as the necessary number of cyclic loadings for double amplitude of axial strain to reach 5%. In this way, stress ratio and $N_l$ can be determined for each specimen.

As a rule, 3 or 4 specimens were taken at each given depth, so that 3 or 4 sets of data for each depth was made available. Figure 4 shows that cyclic undrained triaxial strength is most commonly defined as the stress ratio value that yields a 5% strain obtained on the application of the 20th cyclic load. This paper uses the same definition (see Figure 4). In order to plot a smooth curve, it is, of course, necessary to provide a sufficient spread between
$N_f$-values. In general, $N_f$ frequencies should be obtained with sufficient spread between values, within a range of 100 cycles. Cyclic undrained triaxial strength as determined according to this definition will henceforth be referred to as $R_{L_{20}}$.

![Graphs showing cyclic axial load, cyclic axial displacement, and pore water pressure](image)

Fig. 3 Typical record of wave form

![Graph showing definition of $R_{L_{20}}$](image)

Fig. 4 Definition of $R_{L_{20}}$

6 GENERAL RELATIONSHIP OF TEST RESULTS TO GEOTECHNICAL INDEX VALUES

What soil characteristics can be found to show a basic relationship with cyclic undrained triaxial strength $R_{L_{20}}$? In pursuit of such relationships, the authors first considered the following geotechnical index value of soil:
Fig. 5  Relationship between $R_{H20}$ and $\sigma_v'$

Fig. 6  Relationship between $R_{H20}$ and $N$-value

Fig. 7  Relationship between $R_{H20}$ and $\gamma_d$

Fig. 8  Relationship between $R_{H20}$ and $D_r$

Fig. 9  Relationship between $R_{H20}$ and $D_{10}$

Fig. 10 Relationship between $R_{H20}$ and $D_{10}$

Fig. 11  Relationship between $R_{H20}$ and $F_C$

Fig. 12  Relationship between $R_{H20}$ and $U_C$
\[ \sigma' = \text{in situ effective overburden pressure} \]
\[ N = \text{number of blow counts in the standard penetration test} \]
\[ \gamma_d = \text{dry density of specimen during testing} \]
\[ D_r = \text{relative density of specimen during testing} \]
\[ D_{50} = \text{mean diameter} \]
\[ D_{10} = \text{diameter of grain corresponding to 10\% of grain-size accumulation curve} \]
\[ FC = \text{fine content} \]
\[ U_c = \text{uniformity coefficient} \]

Figures 5 through 12 show the relationships between \( R_L_{20} \) and these constants. The black circles in these graphs represent samples in which \( 30\% \leq FC \leq 60\% \), while the white circles represent samples in which \( FC < 30\% \). These graphs were plotted on the basis of a total of 142 samples (each sample consisting of 3 or 4 specimens). Some \( D_r \) values were not measured. Also, some values for \( D_{10} \) and \( U_c \) were undeterminable.

In each of these graphs, there is a great deal of scatter, so that no clear tendency can be discerned. However, somewhat of a negative correlation between \( R_L_{20} \) and \( \sigma' \) can be seen.

Two major reasons can be cited to explain why a greater relationship between \( R_L_{20} \) and the various geotechnical index values cannot be discerned. First, although there is generally considered to be a strong relationship between cyclic undrained triaxial strength and such structural characteristics of soil as configuration, shape and orientation of grain, no index values that adequately express these characteristics exist.\(^4\) Such techniques as microphotography, x-rays, etc. are frequently used to investigate these characteristics, but these techniques cannot be said to constitute simple soil investigation methods, and were not used in the present research.

We must conclude that the above coefficients alone are not enough to completely explain cyclic undrained triaxial strength.

The second factor is that due to the presence of numerous influential factors, merely finding an overt relationship between \( R_L_{20} \) and other independent variables does not constitute establishing a good correlation.

This paper considers the above point in its use of multiple regression analysis to extrapolate relationships between these constants to find a suitable experimental formula.

7 DERIVATION OF EXPERIMENTAL FORMULA
BY MULTIPLE REGRESSION ANALYSIS

(1) The Multiple Regression Analysis Method

This method will be very simply touched on here. If one has a variable, \( Y \) whose changes can be thought to be explained by a number of variables, \( X_i \), \( Y \) may be expressed by the following formula:

\[ Y = \beta_0 + \Sigma \beta_i X_i \tag{2} \]

The method of statistically determining the most appropriate values for \( \beta_0 \) and \( \beta_i \) is called multiple regression analysis. In the formula, \( Y \) is a dependent variable, \( X_i \) represents independent variables, \( \beta_0 \) is the intercept, and \( \beta_i \) represents multiple regression coefficients. In this paper, \( Y \) is expressed as \( R_L_{20} \) and \( X_i \) is geotechnical index values. The usefulness of the multiple regression analysis formula may be judged according to the values obtained for multiple correlation coefficients and variance ratio. The significance of the multiple regression coefficient can be judged by comparing \( T \)-value with \( T \)-value distribution.
Variables Used in Calculation

The dependent variable used, of course, was $Rl_{20}$. As mentioned in section 6, independent variables that could be used include $\sigma\nu$, $N$-value, $Dr$, $D_{50}$, $D_{10}$, $FC$, $Uc$, etc. However, using all these variables would have only resulted in complicating calculations to no particular advantage. Thus, the independent variables used were limited according to the following specifications:

* Values which could not be obtained by simple methods were eliminated.
* Values which sometimes cannot be determined according to their own definition were eliminated.

Strictly speaking, $\sigma\nu$ cannot be accurately determined without samples being taken in each layer. However, in actual practice, a great deal of sampling is taken in alluvium clayey soil, while in sandy soil, unit weight may be inferred without too great an error, even without sampling. Consequently, if ground water shows hydrostatic pressure distribution, determination of $\sigma\nu$ is an easy matter.

$N$-value is pretty much of an indispensable value in soil exploration, and is also easy to obtain.

Since $r_a$ and $Dr$ require dry density values of samples, undisturbed sand sampling is required to determine these values. Essentially, the object of undisturbed sand sampling is to take samples for cyclic triaxial testing. The cost is high. Thus, it is not appropriate to use this method only for determination of dry density. $D_{50}$ can be easily obtained by mechanical analysis. $D_{10}$ can also be simply obtained by mechanical analysis, but there are some cases when the value cannot be determined. $FC$ can always be determined by mechanical analysis. As with $D_{10}$, $Uc$ can usually be determined by mechanical analysis, but there are some cases when the value cannot be determined.

On the basis of the above reasoning, the independent variables to be used in multiple regression analysis were limited to $N$-value, $\sigma\nu$, $D_{50}$ and $FC$. However, when the correlation coefficients between each of the independent variables were obtained (see Figure 13) it was found that $D_{50}$ and $FC$ show a close relationship in comparison with the other variables. Thus, it was decided not to consider both these values as indispensable variables at the same time. It was also decided to analyze the logarithms of all these values, including $Rl_{20}$.

![Correlation Coefficient between variables](image)

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Fig. 13 Correlation coefficient between variables
Table 2 shows the possible combinations of variables that could be solved for in multiple regression analysis. First, the authors solved for all combinations using $R_{120}$. However, it was decided to solve for only what was considered to be the 3 most suitable combinations using $R_{120}$ as a logarithmic expression.

(3) Calculation Results

Figure 14 shows the results of calculating the different variables combined with $R_{120}$ (as shown in Table 2). The results are multiple correlation coefficients and variance ratios. Those combinations marked with an $x$ represent either one or more multiple regression coefficients whose significance level does not amount to 1%. Those combinations marked with ①, ② and ③ are combinations of relatively high value multiple correlation coefficients that have not been eliminated as having a low significance level. In addition, each variance ratio of ①, ② and ③ surpass the F-value calibration 1% level. Multiple regression analysis was carried out for each of these three combinations, with $R_{120}$ as a logarithmic expression. Figure 15 shows the results. In addition, calculation was carried out for samples whose fine content was less than or equal to 60% (all samples) and for those whose fine content was less than or equal to 30%. In each case, the multiple correlation coefficient was largest when the independent variables were taken as log $\sigma'$, N-value and log FC. Furthermore, the use of $R_{120}$ in the form of a logarithmic expression most accurately expresses experimental results.

Thus, the following formula was obtained.

$$R_{120} = 0.164 \cdot (1.012)^N \cdot (\log FC)^{0.086} \cdot \sigma'^{-0.443} \quad (FC \leq 60\%)$$  (3)

$$R_{120} = 0.159 \cdot (1.012)^N \cdot (\log FC)^{0.093} \cdot \sigma'^{-0.468} \quad (FC \leq 30\%)$$  (4)

Fig.14 Results of multiple regression analysis

Fig.15 Results of multiple regression analysis
8 DISCUSSION

Figure 16 shows the correspondence between the values calculated for $R_L_{20}$ using Formula (3) and actually measured values. These calculated values were obtained using Formula (3). The graph also shows previously available data results, 5, 6, 7.

In figure 16, both ordinate and abscissa are logarithmic expressions. These graphs show a good deal of scatter, but fairly good correspondence.

The exact range of applicability of the independent variables of Formula (3), as determined by conditions at time of experimentation are as follows:

$$\sigma' : 0.15 \leq \sigma' \leq 1.9 \quad \text{kg/cm}^2$$

$$N : 1 \leq N \leq 39$$

$$FC : 0.5 \leq FC \leq 53 \quad \%$$

Other attempts have been made to establish a connection between geotechnical index values and $R_L_{20}$. The following formula is representative of these: 3)

$$R_L_{20} = 0.0882 \sqrt{\frac{N}{\sigma' + 0.7}} - 0.225 \log \frac{D_{90}}{0.35} \quad (0.04 \leq D_{90} \leq 0.6 \text{mm})$$

$$R_L'_{20} = 0.0882 \sqrt{\frac{N}{\sigma' + 0.7}} - 0.05 \quad (0.6 \leq D_{90} \leq 1.5 \text{mm})$$

(5)

Let us consider the meaning of this formula. First, the cyclic undrained shear stress of sand and the relative density of a specimen are said to show the following relationship: 3)

$$R_L_{20} = 0.0042 \cdot \Delta r$$

(6)

Using the research results of Gibbs and Holtz, Meyerhef derived the following formulas: 8, 9)

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**Fig. 16** Comparison between $R_L_{20}$-measured with $R_L_{20}$-calculated

**Fig. 17** Comparison between $R_L_{20}$-measured with $R_L_{20}$-calculated using formula (5)
\[ D_{r^*} = 21 \sqrt{\frac{N}{\sigma' + 0.7}} \] (7)

\( D_{r^*} \) and \( Dr \) do not always show agreement. However, if we tentatively substitute \( D_{r^*} \) for \( Dr \), Formula (6) changes to the following:

\[ R_{I_{50}} = 0.0882 D_{r^*} \] (8)

Formula (8) is determined by effective overburden pressure and \( N \)-value. It will be noted that \( N \)-value is greatly influenced by grain size distribution. Thus a correction term is used, such as the second term on the right side of Formula (6).

Figure 17 shows the correspondence between the actual \( R_{I_{50}} \) value and what it should be according to Formula (5). The graph shows much scattering with no recognizable pattern.

Taking the above into consideration, we may conclude that, as a formula based on a combination of results from sand sampling and cyclic undrained triaxial testing to express cyclic undrained shear strength, Formula (3) is superior to Formula (5). However, on the other hand, the dependency of Formula (3) on \( N \)-value is relatively small, so that even if \( N \)-value changes from 5 to 20, shear strength increases only by a factor of 1.2. This appears to be a small value, and if accurate, shows that \( N \)-value has little relationship with ground improvement.

From a completely different point of view, if it is true that cyclic undrained shear strength has a close relationship with \( N \)-value, we must begin to doubt the accuracy of the results obtained using the cyclic undrained triaxial testing system with sand sampling. We may consider as one possible reason error in sand sampling. In sand sampling, sand that is dense becomes loose, while loose sand becomes dense. Consequently, the difference as expressed as cyclic undrained triaxial strength is small.\(^{10}\) Also, the formula reported herein depends greatly within a restricted range on \( \sigma' \). This has to do with error resulting from such factors as friction of the piston within the cell, effects of the rubber membrane and precision of the loadcell and LVDT.

In any case, there is a necessity for continued research to improve sand sampling, laboratory liquefaction testing and the equipment used therein. In addition, many-faceted research, including use of ground models, aimed at developing methods for evaluating liquefaction strength at the site and exploring the phenomenon of liquefaction are necessary.

REFERENCES

   (In Japanese)


砂質土の繰り返し非排水三軸強度と土質定数の関係

横田耕一郎・今井常雄

概要

（目的）
地震時に地盤内に発生するせん断応力や地盤の保持している繰り返し非排水せん断強度を調べたり、その両者を比較する手法は良く知られている。これらの値は、地盤の液状化評価をする際には必要とされる。これらの値を決める方法は多くのものがあるが、最も精度が高いと考えているものは、発生せん断応力に対しては地震時の地盤の応答計算であり、地盤の繰り返し非排水強度に対しては室内繰り返し非排水三軸強度である。本報告の目的は、繰り返し非排水三軸強度（以下 $R_{20}$ と呼ぶ）の評価をするための有用な実験公式を作成することにある。

実験式における $R_{20}$ は、他の土質力学的定数を基礎として表現される。筆者等の新しい実験公式は、非常に多数多くの繰り返し非排水三軸強度のデータより得られたものであり、公式化するにあたっては、以下に示すような重回帰理論を用いた。

$$R_{20} = \beta_0 + \sum \beta_i X_i$$

ここに

$\beta_0$: 切片  $\beta_i$: 重回帰係数  $X_i$: 土質定数

（要約）
実験には軸載荷型の繰り返し三軸試験装置が用いられた。図-1および図-2には、それぞれ装置の概略図とセルの詳細図を示す。実験に用いた不擾乱試料は、沖積層あるいは埋立の砂層から、サンドサンプリングにより採取されたものである。すべての試料は採取後乾燥されている。表-1には、実験に用いた試料を示す。

図-3には、実験の代表的な波形記録の例を示す。波形記録は基本的には以下の3種類である。

a 繰り返し軸荷重
b 繰り返し軸変位
c 過剰間隙水圧

図-4は、$R_{20}$ の定義を図示したものである。$R_{20}$ は20回の正弦波軸加振を供試体に与えたとき、供試体の軸歪曲率を5%に達しだめるような応力比（繰り返し軸応力初期有効拘束圧の2倍で除した値）として定義される。

筆者等は、当初、$R_{20}$ を表す土質定数としてつぎの量を考えた。

a 有効上載圧 $\sigma'$
b 乾燥密度 $\tau_a$
c 相対密度 $D_r$
d 平均粒径 $D_{50}$
e 10%粒径 $D_0$
f 細粒含有率 $FC$
g 標準貫入試験の打撃回数 $N$

しかしながら、データの入手、評価の難しいものを除

試料数は総計142個であり、それぞれの深度につき1
試料（1試料は3-4供試体）が採取された。

図-5〜図-12に、$R_{30}$ と各種の土質定数との関係を
示す。いずれの図もよい関係を示していない。そこで、
重回帰分析の手法を用いることにした。

図-13は重回帰分析に用いた各変数間の相関係数を示
したものである。当然のことではあるが、$FC$ と $D_{50}$
の相関係数が大きいのが目立つ。これら2つの変数は重
相関式において同時に考慮することのないようにした。

表-2には、以上の検討結果より考え得るすべての重
回帰分析の組合せを示す。$R_{30}$ を用いた重回帰分析はす
べての組合せについて行なったが、一方、log $R_{30}$ を用
いた重回帰分析は、統計学的に有効と考えられる3組の
みについて行なった。（図-14の①〜③および図-15参
照）

$R_{30}$、log $R_{30}$ に対する計算結果を、重回帰関係およ
び分散比の形にして、それぞれ図-14および図-15に
示す。これらの計算結果より、最も重回帰関係の高いも
のは、log $R_{30}$ と log $\sigma'$、$N$、$FC$ を組合せた場合である

ことがわかる。その実験式を示せば、つぎのようである。

\[ \log R_{30} = -0.785 + N \cdot \log 1.012 + 0.0868 \log FC - 0.441 \log \sigma' \]

すなわち、$R_{30}$ は

\[ R_{30} = 0.164 \cdot (1.012)^N \cdot (FC)^{0.0868} \cdot (\sigma')^{-0.441} \] 本文の式(3)

図-16には式(3)を用いて計算した $R_{30}$ （計算 $R_{30}$）
と実測した $R_{30}$ （実測 $R_{30}$）の比較を示す。図には、既存
データを用いた両者の比較をも示してある。両者の関係
はかなり良いことがわかる。

図-17には、既存の実験式（本文の式(6)）を用いて求
めた計算値 $R_{30}$ を使い、図-16と同様の整理をしたも
のを示す。この図では、一般的傾向は認められない。式(5)
と式(3)の大きな違いは、両式の $N$ 値と $\sigma'$ に関する依存
の程度である。今日までの多くの研究では、きれいな砂
の場合、繰り返し非排水せん断強度は$N$値に大きく影響
されているといわれている。この観点から考えると、$N$値と
繰り返し非排水せん断強度の関係においては、(5)式の方
が従来からの見解とよく一致するようである。しかしながら、
サンドサンプリングと、繰り返し非排水三軸試験
の組合せより評価される、繰り返し非排水せん断強度を
決定するための実験式という観点からは、(3)式の方が優
れているようである。

別の観点に立って、式(3)は、繰り返し非排水せん断
強度の評価に関して、現在用いられているシステムの精度
に疑問を投げかけるものともいえる。この分野の今後の
よりいっそうの研究が待たれる。